

Coherence vortices in one spatial dimension

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Coherence vortices are screw-type topological defects in the phase of Glauber's two-point degree of quantum coherence, associated with pairs of spatial points at which an ensemble-averaged stochastic quantum field is uncorrelated. Coherence vortices may be present in systems whose dimensionality is too low to support spatial vortices. We exhibit lattices of such quantum-coherence phase defects for a one-dimensional model quantum system. We discuss the physical meaning of coherence vortices and propose how they may be realized experimentally.

PACS numbers: 03.75.-b, 42.25.Kb

I. INTRODUCTION

Vortices have fascinated the minds of scientists throughout history. Beginning with the angular momentum eigenstates of the hydrogen atom, quantum-mechanical vortices emerged as key entities characterizing quantum liquids such as superconductors and superfluid helium [1]. More recently, quantized vortices have been observed e.g. in Bose-Einstein condensates [2], quantum degenerate Fermi gases [3], and in coherent optical [4] and acoustic fields [5]. Quantized vortices, known as coherence vortices, have since been discovered in the cross-spectral density and related coherence functions of partially-coherent classical-optical fields [6–14].

Here we show that coherence vortices, which are screw-type singular phase defects in Glauber's second order degree of quantum coherence, may exist even in systems with only one spatial dimension where orbital angular momentum and conventional quantized vortices cannot be defined. We exhibit a one-dimensional model system in which decoherence [15, 16] is accompanied by a proliferation of quantized phase vortices in the coherence function associated with the resulting statistical mixture. In a space of such low dimensionality, the presence of a statistical mixture is a necessary condition for the existence of coherence vortices. We outline proposals for experimentally creating and observing coherence vortices and vortex lattices in spatially one-dimensional systems, and speculate the possible connection between coherence vortices and decoherence.

II. QUANTUM VORTICES

Vortical flows, associated with velocity fields $\mathbf{v}(\mathbf{r}, t)$, have non-zero circulation

$$\Gamma = \oint_{\Omega} \mathbf{v}(\mathbf{r}, t) \cdot d\mathbf{l}. \quad (1)$$

Here, Ω denotes a smooth closed contour which is traversed once in a positive sense at a fixed time t , $d\mathbf{l}$ is an infinitesimal arc length vector pointing along the direction in which Ω is traversed, and \mathbf{r} is position. For single-valued differentiable pure-state complex scalar

wave-functions $\psi(\mathbf{r}, t)$ with phase $\varphi(\mathbf{r}, t) = \arg[\psi(\mathbf{r}, t)]$ we can define a velocity field

$$\mathbf{v}(\mathbf{r}, t) = \frac{\hbar}{m} \nabla \varphi(\mathbf{r}, t), \quad (2)$$

where m is the mass of a particle. The corresponding circulation integral is quantized in units of \hbar/m :

$$\Gamma = \frac{\hbar}{m} \oint_{\Omega} \nabla \varphi(\mathbf{r}, t) \cdot d\mathbf{l} = n \frac{\hbar}{m}, \quad (3)$$

where the integer n is termed the topological charge. This quantization of circulation is a consequence of the single-valued and continuous nature of the function $\psi(\mathbf{r}, t)$ which may describe e.g. an optical or a matter-wave field. The integer n denotes the number of circulation quanta and is related to the orbital angular momentum of the particles which is only meaningful in two or more spatial dimensions.

Let us now shift attention, from pure states to mixed states. If one has a mixed-state wave-function, describing an ergodic stochastic quantum system via an ensemble of realizations each of which carry a specified statistical weight, vortical flows may be studied in terms of the circulation integral in Eq.(1) applied to the velocity field $\mathbf{v}_{av}(\mathbf{r}) = \langle \mathbf{v}(\mathbf{r}, t) \rangle$, where angular brackets denote ensemble averaging (obtained, for example, using the density-matrix formalism). We assume there to be no explicit time-dependence in the external parameters and thus the time-independence of $\mathbf{v}_{av}(\mathbf{r})$ follows from the ergodicity of the stochastic process describing the field. The circulation of the averaged velocity field $\mathbf{v}_{av}(\mathbf{r})$ is not quantized in general, even though circulation of the velocity field is quantized for individual members of the ensemble.

However, as shown below, quantized circulations exist for *coherence functions* associated with stochastic quantum fields. Moreover, such quantized coherence circulation may be meaningfully defined for fields in one spatial dimension, even though this dimensionality is too low to permit spatial vortical flows. Lastly, we show that, for the one-dimensional model system studied here, a proliferation in the number of coherence vortices and their associated coherence-function zeros is associated with the process of decoherence.

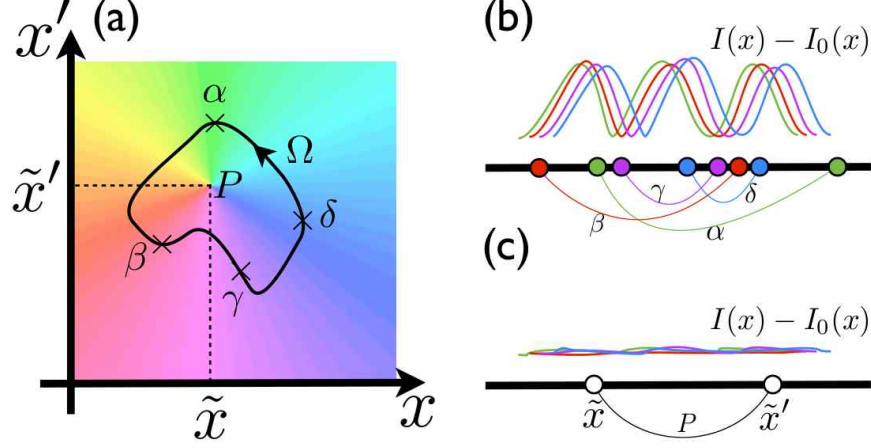


FIG. 1. (Color online) A coherence vortex as a double-slit interference ratchet. (a) The presence of a coherence vortex at point $P = (\tilde{x}, \tilde{x}')$ is revealed via non-zero circulation p in Eq. (5). Four coordinate pairs on the contour Ω are labelled α, β, γ , and δ . (b) In general, non-zero fringe visibility will result should a Young-type interference experiment be performed by combining the disturbances from the pair of points corresponding to α ; the intensity of the resulting pattern is sketched as a solid (green) line. Interference patterns are also sketched to correspond to the coordinate pairs β (red), γ (magenta) and δ (blue). As one cycles from $\alpha \rightarrow \beta \rightarrow \gamma \rightarrow \delta \rightarrow \alpha$, the corresponding series of Young-type fringes ‘ratchets’ through one cycle. (c) If a Young-type interference experiment were performed by combining the disturbances from the pair of points $P = (\tilde{x}, \tilde{x}')$, corresponding to the core of the coherence vortex, zero fringe visibility would result since $g^{(1)}(\tilde{x}, \tilde{x}') = 0$ (see Eq. (4)).

III. COHERENCE VORTICES

For an ergodic stochastic one-dimensional complex quantum field $\Psi(x, t)$ the normalized two-point equal-time correlation function is given by the following special case of Glauber’s degree of quantum coherence [17]

$$g^{(1)}(x, x') = \frac{\langle \Psi^\dagger(x, t) \Psi(x', t) \rangle}{\sqrt{|\langle \Psi(x, t) \rangle|^2 |\langle \Psi(x', t) \rangle|^2}}, \quad (4)$$

where the angular brackets now denote both quantum mechanical and statistical average. Note that $g^{(1)}(x, x')$ is independent of time, on account of the assumed ergodicity of the stochastic process.

As a consequence of the single-valued and continuous nature of each complex field in the ensemble, it follows that $g^{(1)}(x, x')$ is also a continuous single-valued complex function of its arguments. Hence, for any simple smooth positively-traversed closed loop Ω in the two-dimensional space coordinatized by (x, x') , which is such that $|g^{(1)}(x, x')|$ is strictly positive for all $(x, x') \in \Omega$, the coherence-function circulation integral will be given by

$$\oint_{\Omega} \nabla_{\perp} [\arg g^{(1)}(x, x')] \cdot d\mathbf{l}_{\perp} = 2\pi p, \quad (5)$$

where \perp denotes two-dimensional operators and vectors, and p is an integer denoting topological charge.

Suppose that, for a given closed contour Ω in the x - x' plane shown in Fig. 1a, p in Eq. (5) has a non-zero value. Then, there exists at least one coordinate pair (\tilde{x}, \tilde{x}') in the interior of Ω , at which $g^{(1)}(x, x')$ vanishes. This

statement may be proved by contradiction, as follows: The continuity of $g^{(1)}(x, x')$ implies that $\arg[g^{(1)}(x, x')]$ will be defined and continuous for every coordinate pair (x, x') inside and on Ω . Now continuously contract the contour Ω to a point, which coincides with some coordinate pair (\tilde{x}, \tilde{x}') in the interior of Ω , via the infinite sequence:

$$\Omega \rightarrow \Omega_1 \rightarrow \Omega_2 \rightarrow \Omega_3 \cdots \rightarrow (\tilde{x}, \tilde{x}') \quad (6)$$

which is such that all “intermediate contours” Ω_1, Ω_2 etc. are fully contained within the region bounded by Ω . Since $\arg g^{(1)}(x, x')$ can be continuously contracted to a point without changing the value of circulation between adjacent members of the contour sequence in Eq. (6), the non-zero circulation about Ω (i.e. $2\pi p$, for some non-zero integer p) is equal to the zero circulation about an infinitesimally-small contour wrapped around (\tilde{x}, \tilde{x}') (this latter circulation vanishes, since $\arg g^{(1)}(x, x')$ is continuous at and in the infinitesimal vicinity of (\tilde{x}, \tilde{x}')). This logical contradiction implies the falsity of the initial supposition that $|g^{(1)}(x, x')| > 0$ for every coordinate pair (x, x') inside Ω . Hence $|g^{(1)}(x, x')|$ vanishes for at least one coordinate pair (\tilde{x}, \tilde{x}') inside Ω .

Non-zero coherence current circulation $2\pi p$ is associated with quantized coherence vortices, as a low-dimensional special case of the concept of coherence vortices formulated in the theory of partially-coherent classical-optics fields, as quantified through the mutual coherence function or cross-spectral density [6–14]. In light of the theorem proved above, non-zero p is associated with at least one pair of points (\tilde{x}, \tilde{x}') for which $g^{(1)}(x, x')$ vanishes. Notwithstanding that the term “co-

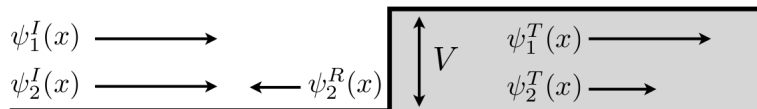


FIG. 2. Schematic for coherence vortex lattice experiment. Waves described by $\psi_1(x)$ and $\psi_2(x)$ scatter from a potential step of height V . The potential barrier is transparent to the first species whereas the second species is both reflected and transmitted by the potential.

herence vortex” is employed in the literature, we prefer the term “decoherence vortex” since (\tilde{x}, \tilde{x}') is a pair of points at which the two-point correlation vanishes—an absence of coherence—indicating a topologically-inevitable zero in the correlation between two spatially-distinct points in a stochastic quantum field. Having noted this preference, we will retain the term coherence vortex to generically refer to screw-type defects in the phase of complex coherence functions such as Glauber’s two-point correlation function, the mutual coherence function or the cross-spectral density.

The physical meaning of a coherence vortex may be further clarified with reference to Fig. 1. The phase of the complex function $g^{(1)}(x, x')$ is denoted by color in Fig. 1(a) where the core of the coherence vortex at (\tilde{x}, \tilde{x}') is labeled by P . Four locations α, β, γ , and δ corresponding to pairs of points are marked by crosses along the path Ω , which encircles the phase singularity at point P . Figure 1(b) shows a schematic of the interference fringes observed in a Young-type double-slit experiment where the two slits are placed at points determined by α, β, γ , and δ and are illuminated by a spherical wave of light sourced from the corresponding locations x and x' . The background intensity $I_0(x)$ produced by a completely incoherent source field has been subtracted from the total intensity $I(x)$. The resulting interference fringes are seen to ‘ratchet’ as the coherence vortex core is encircled. That is, the troughs and peaks of the interference pattern move continuously in one direction (within the overall fringe envelope) when a loop around the coherence vortex is traversed. If the two slits are placed at the location of the coherence vortex core $P = (\tilde{x}, \tilde{x}')$ the intensity of the fringes disappears, as illustrated in Fig. 1(c).

IV. COHERENCE VORTEX LATTICES IN ONE SPATIAL DIMENSION

A. Experimental systems

Having discussed the elements of coherence vortices in one spatial dimension, we now propose experiments to observe them. Suppose we have a monoenergetic beam of unpolarized photons propagating from negative infinity $x = -\infty$ through vacuum and into a block of birefringent material placed at $x = 0$ which can be viewed as a potential step whose height V depends on the state

of the incoming photons. The beam is composed of two unentangled polarization states of light $|\sigma_+\rangle$ and $|\sigma_-\rangle$ and can be described as a statistical mixture of two pure state wavefunctions $\psi_{\sigma_+}(x)$ and $\psi_{\sigma_-}(x)$. Due to the polarization-dependent propagation in the medium the two components experience different phase evolution and travel through the material with different wave vectors. The interference of the different wave components facilitates the development of coherence vortex structures. Notice that the light source itself must *not* be strictly coherent, as a necessary condition for the existence of coherence vortices in one spatial dimension. By performing a set of interference measurements by varying the locations x and x' of the interferometer arms the phase map of the two-point coherence function $g^{(1)}(x, x')$ may be generated, from which the loci of the coherence vortices may be read off. The photon field could also be replaced with sound waves to observe acoustic coherence vortices in one spatial dimension.

Similarly, we may also consider a matter-wave analog of the electromagnetic field scattering experiment, where the beam of photons would be replaced by, e.g., an electron beam or a stream of cold atoms such as those in an atom laser beam sourced from a Bose-Einstein condensate. In the case of electrons, the spin-up $|\uparrow\rangle$ and spin-down $|\downarrow\rangle$ states of the electrons provide the source of the required mixed states and an external magnetic field could be used for creating the polarization dependent potential. In the case of an atomic beam two or more internal hyperfine spin states would facilitate the mixing of states and the required potential could be obtained using optical or magnetic coupling to the atomic dipole moments. In both cases the measurement of coherence vortices could be achieved via a two-point interference experiment.

B. Theoretical model

Turning to the theoretical description of the above and related physical systems, consider a mixed quantum state composed of two ensemble members such that

$$g^{(1)}(x, x') = \psi_1^*(x)\psi_1(x') + \psi_2^*(x)\psi_2(x'), \quad (7)$$

less normalization. For incoming plane-wave states, see Fig. 2, which scatter from a potential step of height V transparent to the first species $\psi_1(x)$, solving the

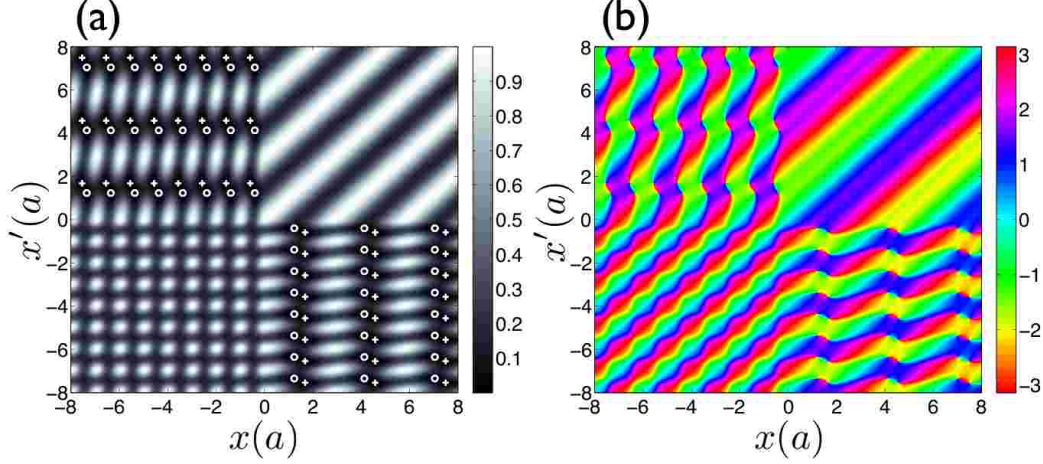


FIG. 3. (Color online) Stationary interlaced coherence vortex-antivortex dipole lattice. (a) Coherence density $|g^{(1)}(x, x')|^2$ with the peak density normalized to unity; (b) coherence phase $\arg[g^{(1)}(x, x')]$. Locations of coherence vortices (coherence antivortices) are marked in frame (a) by crosses (circles). The unit of length $a = 1/k$ and $V/E = 0.99$.

Schrödinger equation we obtain

$$g^{(1)}(x < 0, x' > 0) = e^{i(-kx+kx')} + T(k, q)e^{i(-kx+qx')} + T(k, q)R(k, q)e^{i(kx+qx')}, \quad (8)$$

where k and q are the wavevectors on each side of the potential step located at $x = 0$ and

$$\begin{aligned} T(k, q) &= 2k/(k + q), \\ R(k, q) &= (k - q)/(k + q) \end{aligned} \quad (9)$$

are the transmission and reflection coefficients, respectively. The energy $E > V$ of the incoming particles of mass m and the height of the potential step V are linked to the wave vectors by the formulae

$$\begin{aligned} k^2 &= 2mE/\hbar^2, \\ q^2 &= 2m(E - V)/\hbar^2. \end{aligned} \quad (10)$$

Phase singularities can be produced by an interference of three or more plane waves [18–23]. Guided by this fact and using complex phasors [19] we solve for the zeroes of $g^{(1)}(x, x')$ in regions $xx' < 0$, to find the locations of the quantized coherence vortices. We obtain two interlaced lattices determined by lattice vectors (x_u, x'_v) :

$$\begin{aligned} x_u &= p \frac{\beta - \alpha}{2k} - u \frac{\pi}{k}, \\ x'_v &= p \frac{-\alpha}{k - q} + v \frac{2\pi}{k - q}, \end{aligned} \quad (11)$$

where u and v are integers, $x_u < 0$ and $x'_v > 0$ and the Cartesian shifts are determined by the angles

$$\begin{aligned} \alpha &= \pi/p - \arccos(1 + T^2 - T^2 R^2 / 2T), \\ \beta &= \pi/p + \arccos(1 - T^2 + T^2 R^2 / 2TR). \end{aligned} \quad (12)$$

These two lattices correspond to, respectively, coherence vortices and coherence antivortices, determined by the

sign of the coherence circulation $p = \pm 1$. In Fig. 3(a) we have plotted the coherence density $|g^{(1)}(x, x')|^2$ scaled by its maximum value. The coherence vortices and antivortices have been marked by circles and crosses, respectively, at locations determined by Eq. (11). Figure 3(b) shows the corresponding phase plot $\arg[g^{(1)}(x, x')]$ from which it can be verified that the loci of phase singularities correspond to the cores of the coherence vortex lattice. The Hermitian property of $g^{(1)}(x, x')$ is inherited by the coherence vortex lattice. Numerically, it is possible to construct the function $g^{(1)}(x, x')$ for an arbitrary potential shape and to find the resulting coherence vortices and antivortices. For such general potential landscapes, the coherence vortices and antivortices will not lie on a regular lattice. We emphasize that if the density matrix for one-dimensional systems can be written in a pure-state form, coherence vortices are absent. This follows from the fact that we need a minimum of three phasors to produce phase singularities via plane-wave interference.

V. DISCUSSION

Conventionally, quantized vortices have been studied as phase singularities in the “medium” supporting them, e.g. superconducting order parameter, complex electromagnetic field, or macroscopic wavefunction describing a matterwave of degenerate quantum liquids. Here we have considered quantized vortices emerging in the coherence or correlation function derived from the quantum states describing the system. Such phase singularities, known as coherence vortices, are topologically enforced zeroes in the two-point coherence function. They appear as a field interference and hence may emerge even when there are no conventional quantized vortices in the field itself. In particular, coherence vortices can exist in systems with one spatial dimension only, as shown here, where conven-

tional vorticity is manifestly absent. We have described the physics of coherence vortices and have proposed experiments to observe them. We have shown using an analytically soluble model how a light or matter-wave field scattering from a step potential creates a stationary coherence vortex and antivortex lattice.

Decoherence is considered to be a mechanism for the way classical realism emerges from a quantum coherent system description in terms of mixed-state density matrices. We speculate it to be possible that decoherence in a quantum system leaves a topological imprint in terms of coherence vortices, which could be used for detecting the amount of decoherence in a system. This possibility is suggested by the fact that, in the model

one-dimensional system studied here, coherence vortices are absent from the pure-state systems and only emerge when one has at least the two-member statistical mixture given by Eq. (7). Further work is also required to clarify whether the theory of coherence vortices may be connected with the proliferation of thermally activated vortex-antivortex pairs [24] and quasi-coherence in the observed superfluid to normal transition in cold quasi-two-dimensional Bose gases [25–28]. Finally, the presence of coherence vortices might signal some new physical material property in analogy with the way conventional quantized vortices are inherently linked together with the phenomena of superfluidity and superconductivity.

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